MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

TMA1101 – Calculus

(All sections / Groups)

4 MARCH 2020
9.00 – 11.00am
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **5** pages with **5** questions only excluding the cover page.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

4. **No calculators are allowed.**

QUESTION 1 [10 MARKS]

(a) Find the following limits.
[*You must show at least one intermediate step where* $\lim_{x\to c}$ *is still needed.*]

(i) $\lim_{x\to 3}\dfrac{2x^2-5x-3}{x^2-3x}$ [1.5 marks]

(ii) $\lim_{x\to 0}\dfrac{(x^2+x)\sin x}{x^2}$ [1.5 marks]

(b) Given

$$f(x)=\begin{cases}\sin x, & x>\dfrac{\pi}{4}\\ \cos x, & x<\dfrac{\pi}{4}\end{cases}$$

(i) Does $\lim_{x\to\frac{\pi}{4}} f(x)$ exist? Give your reason. If the limit exists, state its value.

(ii) Is f continuous at $\dfrac{\pi}{4}$? Give your reason. [3.5 marks]

(c) (i) State the intermediate value theorem (i.e., the full statement including the hypothesis and the conclusion).

(ii) Show that there is a root of the equation $x^4+2x^2-7=0$ in the interval [0, 2]. You must write proper steps to arrive at conclusion; just writing some calculations would not be enough.

[3.5 marks]

Continued.....

QUESTION 2 [10 MARKS]

(a) Use the formal definition of the derivative to compute $f'(2)$ when $f(x)=x(3x-2)$.

You are reminded to write proper steps.

[2.5 marks]

(b) Find $\frac{dy}{dx}$ with y as given.

[Use the product rule or the quotient rule for differentiation; show proper steps.]

(i) $y=3x\sin^2 x$ [2 marks]

(ii) $y=\frac{x^2-4}{e^{-2x}}$ [2 marks]

(c) The point $(1,2\pi)$ lies on the curve $x^2y+\sin y=2\pi$.

Use implicit differentiation to obtain $\frac{dy}{dx}$ **in terms of x and y.**

Then find the equation of the tangent to the curve $x^2y+\sin y=2\pi$ at the point $(1,2\pi)$.

[3.5 marks]

Continued.....

QUESTION 3 [10 MARKS]

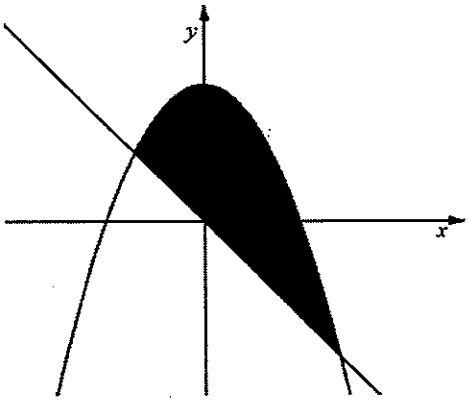
(a) (i) Use $\cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}$ and $\sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i}$ to find the values of A and B which make the equation $\sin 4\theta\cos 3\theta = A\sin 7\theta + B\sin\theta$ an identity.

(ii) Then, evaluate $\int_0^{\frac{\pi}{3}} \sin 4x\cos 3x\, dx$

[3.5 marks]

(b) Use the method of substitution to evaluate $\int 2x^2\sqrt{x^3+8}dx$ [2.5 marks]

(c) The figure shows a region bounded by the parabola $y = 4-x^2$ and the straight line $y = -3x$.



(i) Determine the x-coordinates of the points of intersection between the parabola and the straight line.

(ii) Write down a definite integral that can be used to find the area of this region and proceed to find the area.

[4 marks]

Continued.....

QUESTION 4 [10 MARKS]

(a) Given the sequence $\{a_n\}$ with $a_n = \dfrac{n^2-4n}{3n^2+10n}$.

(i) Determine $\lim_{n\to\infty} a_n$; you are reminded to write proper steps.

(ii) Determine whether the infinite series $\sum_{n=1}^{\infty} \dfrac{1+2n^2}{3n^2+5}$ is convergent. Give the reason for your answer.

[2 marks]

(b) By using the direct comparison test. Determine whether the infinite series

$$3+\frac{3}{\sqrt{2}}+\frac{3}{\sqrt{3}}+\frac{3}{\sqrt{4}}+\frac{3}{\sqrt{5}}+\cdots$$

is convergent.

[2 marks]

(c) Find the Taylor polynomial of order 3 for $f(x)=\dfrac{1}{x-3}$ centered at $a=5$. You are reminded to write proper steps.

[4 marks]

(d) A periodic function $f(x)$ with period 2π is defined as

$$f(x)=x \quad for \quad -\pi \le x \le \pi$$

The Fourier series of $f(x)$ has the form $f(x)=\dfrac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$.

Determine the value of b_3.

[2 marks]

Continued.....

QUESTION 5 [10 MARKS]

(a) Given $F(x,y)=xy^2+\sin x-e^x\ln y$, find the partial derivatives $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$.

[1 mark]

(b) Solve the first order separable differential equation $2y\frac{dy}{dx}+4x=8\frac{dy}{dx}+3$ subject to the initial condition $y(1)=0$. You may leave your answer in ***implicit*** form.

[2.5 marks]

(c) (i) Show that x^2 is an integrating factor for the first order linear differential equation

$$\frac{dy}{dx}+\frac{2y}{x}=x^2$$

(ii) Solve the differential equation and give your solution in ***explicit*** form.

[3 marks]

(d) (i) Find the roots of the characteristics equation of the homogeneous differential equation $y''-8y'+16y=0$. Then write down the complementary function y_h of this homogeneous equation.

(ii) Find the particular equation of the second order differential equation $y''-8y'+16y=5x$.

(iii) Hence, write down the general solution for the differential equation $y''-8y'+16y=5x$.

[3.5 marks]

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